Experimenting from a Distance

In case of optical Fourier-Transformation

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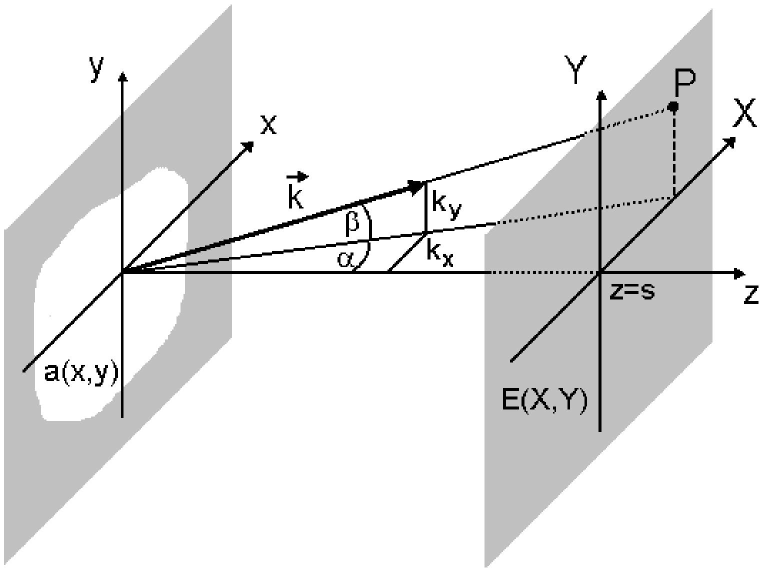
Idea

Diaphragm function a(x,y) describing the diffracting objekt



Plane of diffracting object

Plane of diffracted intensity

$I(X,Y) \sim |E(X,Y|^2$

$E(X,Y) = FT[a(x,y)]$

Intensity distribution in Point P on screen is square of Fourier transformed of diaphragm described by a suited function.

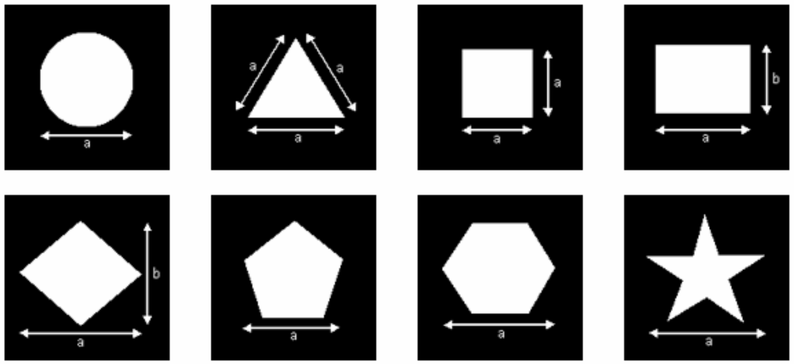
$\Rightarrow$ Use an optical setup to teach/model FT experimentally

FT and basic laws of FT

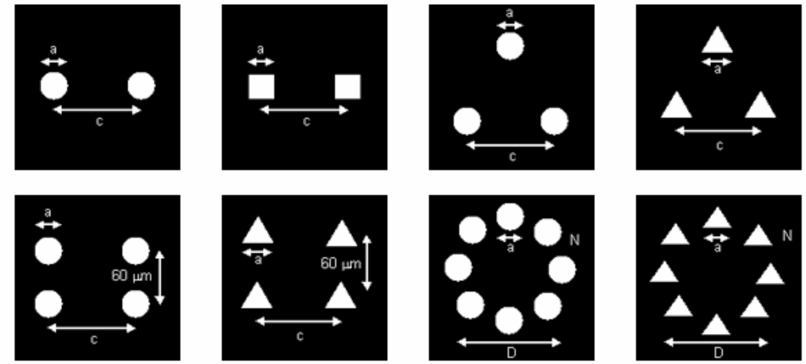
Property	**f(x,y) =**	**F(u,v) = FT[f(x,y)]**
Scaling	$g(ax,by)$	$1/\lvert ab\rvert \cdot G(u/a,v/b)$
Linearity	$a \cdot g(x,y) + b \cdot h(x,y)$	$a \cdot G(u,v) + b \cdot H(u,v)$
Translation	$g(x-x_0,y-y_0)$	$e^{ix_0 u} \cdot e^{iy_0 v} \cdot G(u,v)$
Convolution	$g(x,y) * h(x,y)$	$G(u,v) \cdot H(u,v)$
Inversion of convolution	$g(x,y) \cdot h(x,y)$	$G(u,v) * H(u,v)$
Separation	$g(x) \cdot h(y)$	$G(u) \cdot H(v)$
Inversion of translation	$e^{iu_0 x} \cdot e^{iv_0 y} \cdot g(x,y)$	$G(u-u_0,v-v_0)$

$\Rightarrow$ Aim to visualize mathematical relations

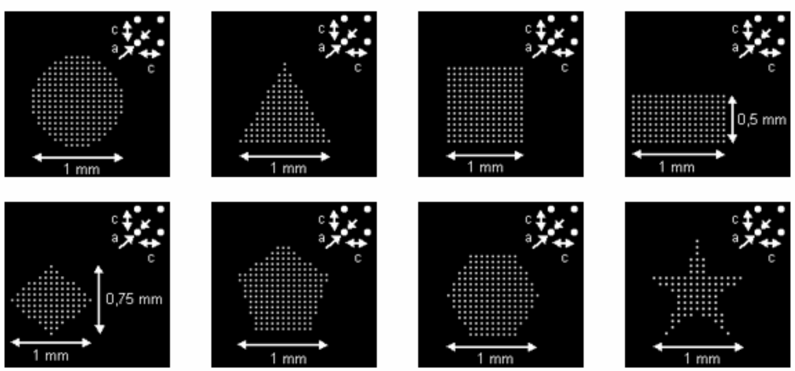
Diffracting objects



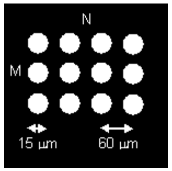
Single forms
(8 objects)



Single forms at corners of forms
(48 objects)

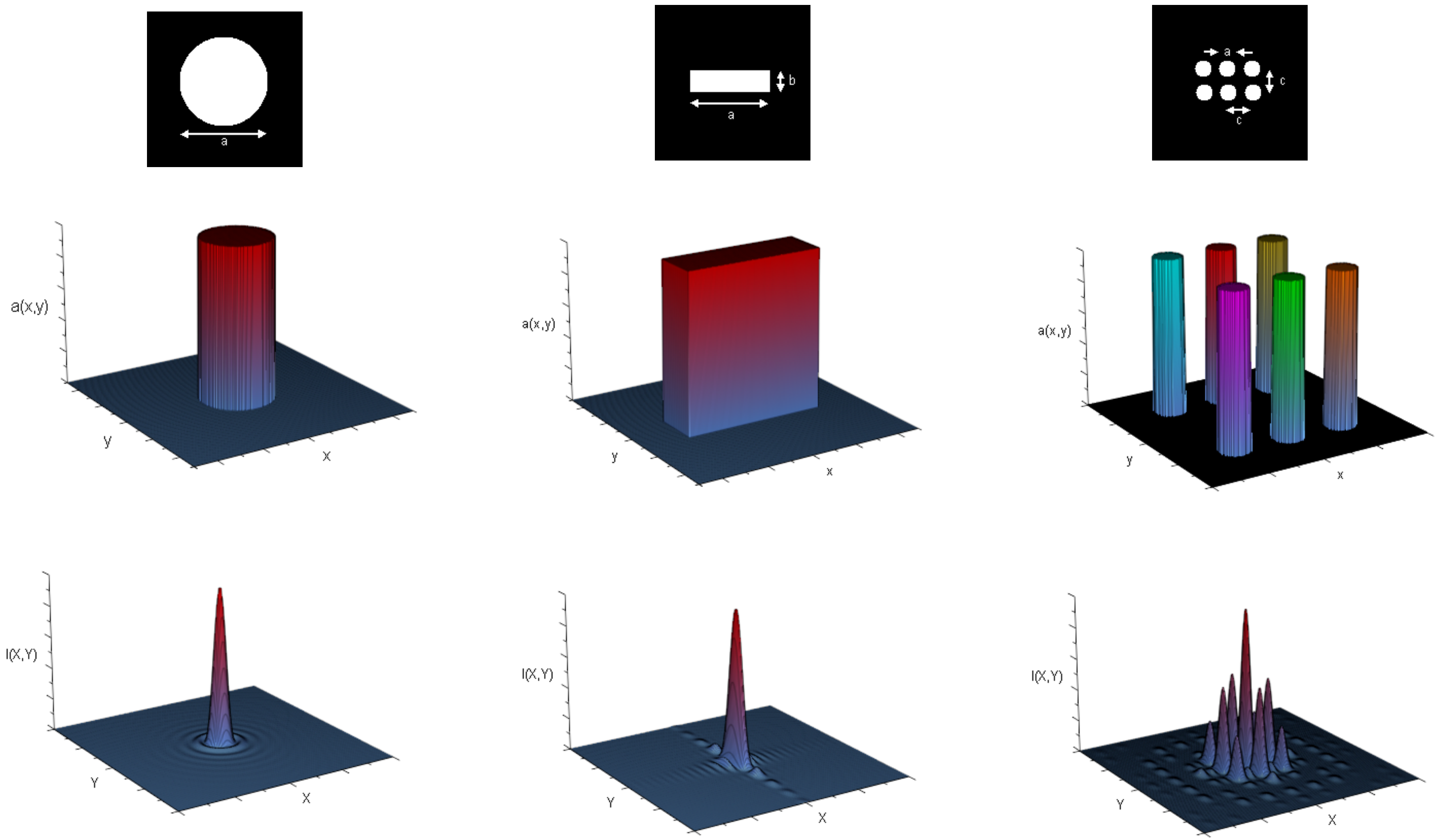


Quadratic grating with circles as single form confined by different single forms
(16 objects)



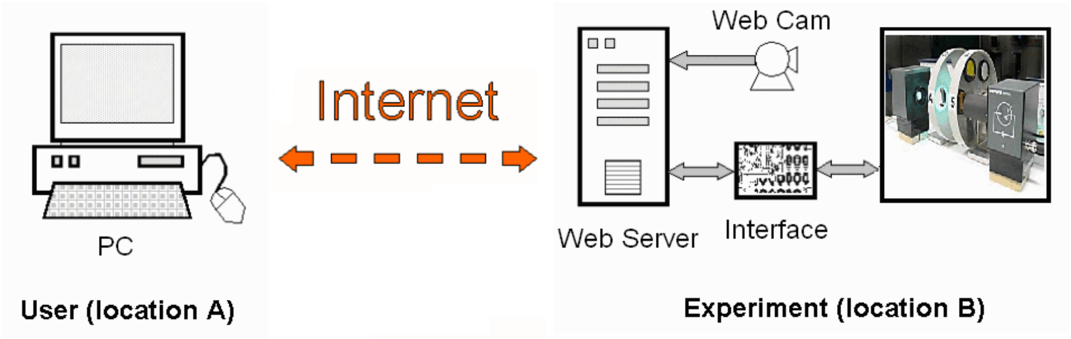
Quadratic N x M – grating with circles as single form
(33 objects)

Some examples

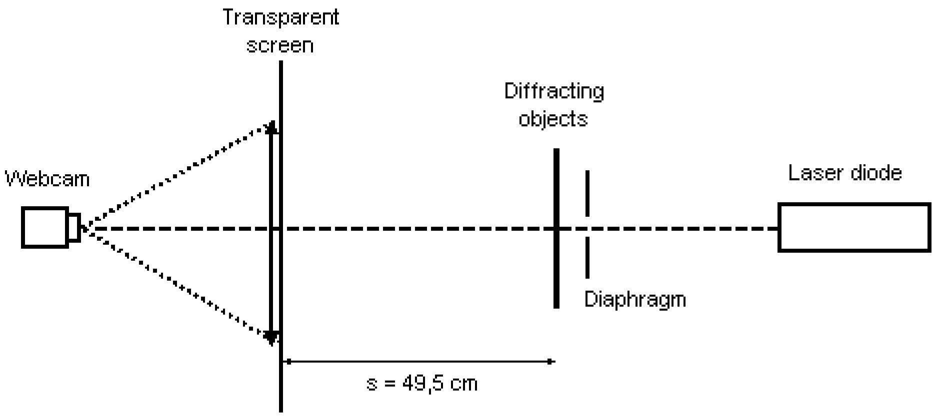


⇒ Check, if theoretically expected and experimentally determined intensity pattern will agree.

RCL-experiment



Principle



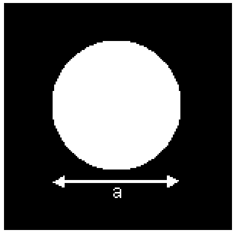
Experimental setup of RCL „Optical Fourier-Transformation“

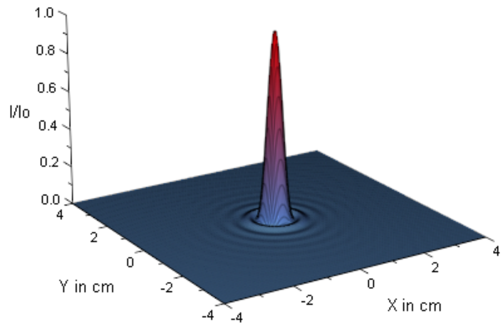
Properties

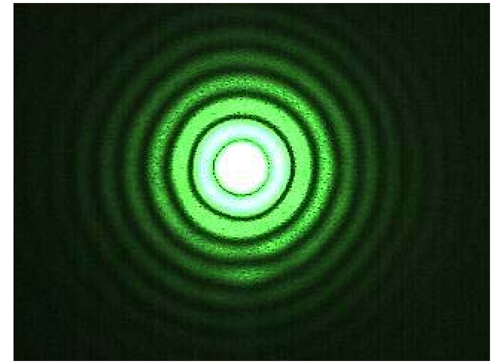
- interactive
- authentic
- autonomous
- robust
- accessible 24 h/7d
- no additional software
- no registration
- no costs to use

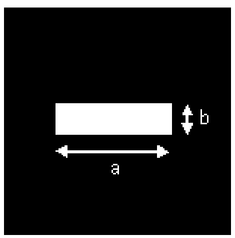
Demonstration of RCL-experiment

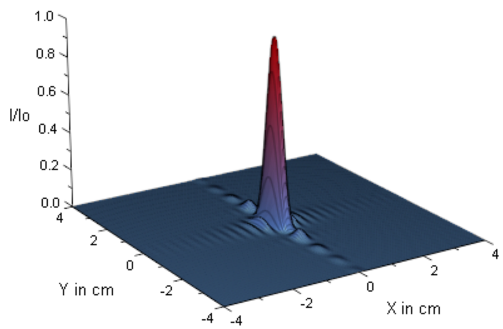
Comparison Experiment - Theory

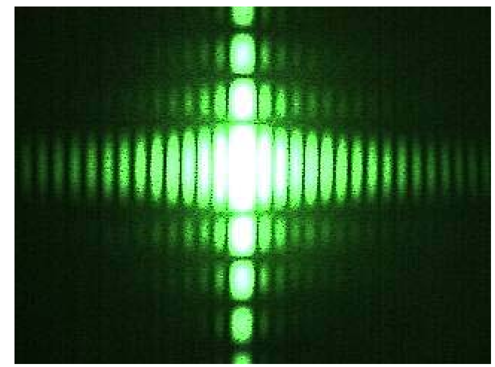


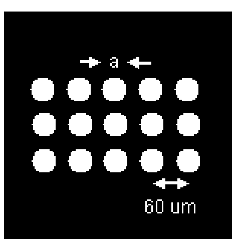


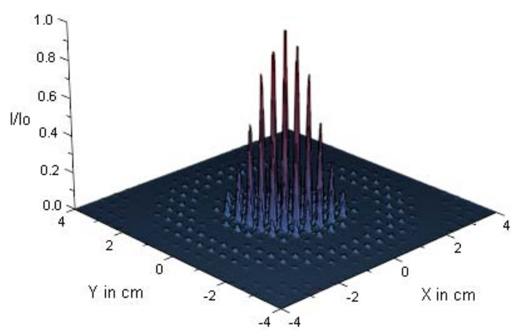


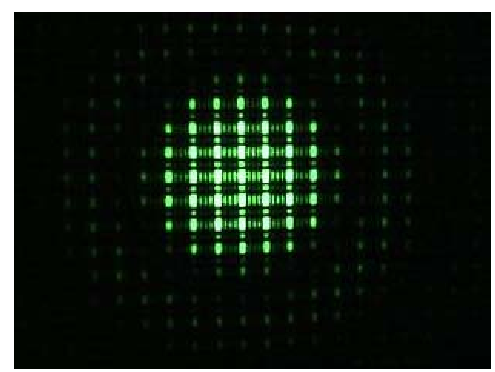




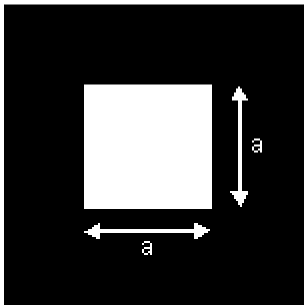




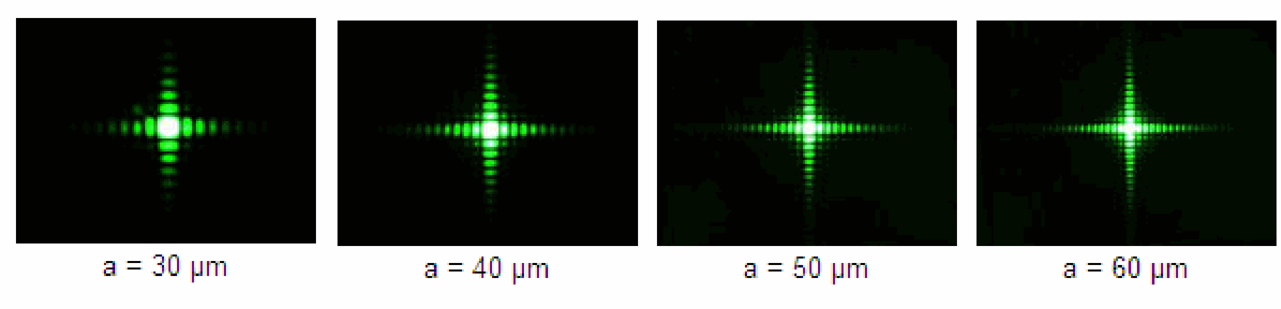




Visualize basic laws of FT - Scaling



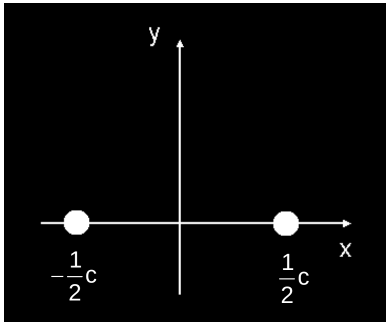
Diffracting object



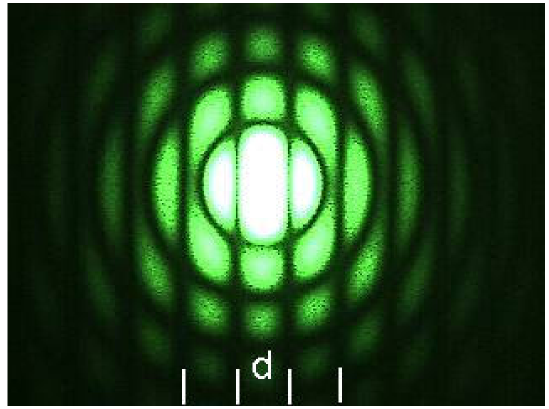
Diffraction pattern

a in µm	Distance d´ between third minima in px	Distance d´ between third minima in cm	ad in 10^{-6}
30	81	4.10	1.22
40	62	3.12	1.25
50	48	2.41	1.21
60	41	2.06	1.24

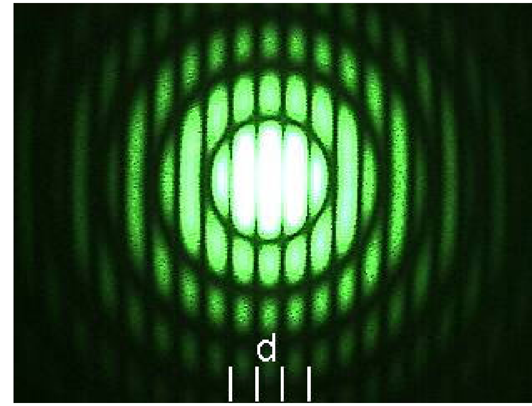
Proof basic laws of FT – Linearity and Translation



Diffracting object
circles distance c



c = 30 μm



c = 60 μm

Diffraction pattern for variable distance c

$$a(x,y) = a_s(x-\frac{c}{2},y) + a_s(x+\frac{c}{2},y)$$

$$\underline{E}(k_x,k_y) = FT[a(x,y)] \overset{\text{Linearity}}{=} FT[a_s(x-\frac{c}{2},y)] + FT[a_s(x+\frac{c}{2},y)]$$

$$\overset{\text{Translation}}{=} e^{-i\frac{c}{2}k_x} \cdot \underline{E}_s(k_x,k_y) + e^{i\frac{c}{2}k_x} \cdot \underline{E}_s(k_x,k_y) = 2 \cdot \underline{E}_s(k_x,k_y) \cdot \cos(\frac{c}{2}k_x)$$

$$d_{theo} = \frac{\lambda s}{c} = 4.4 \text{ mm}$$

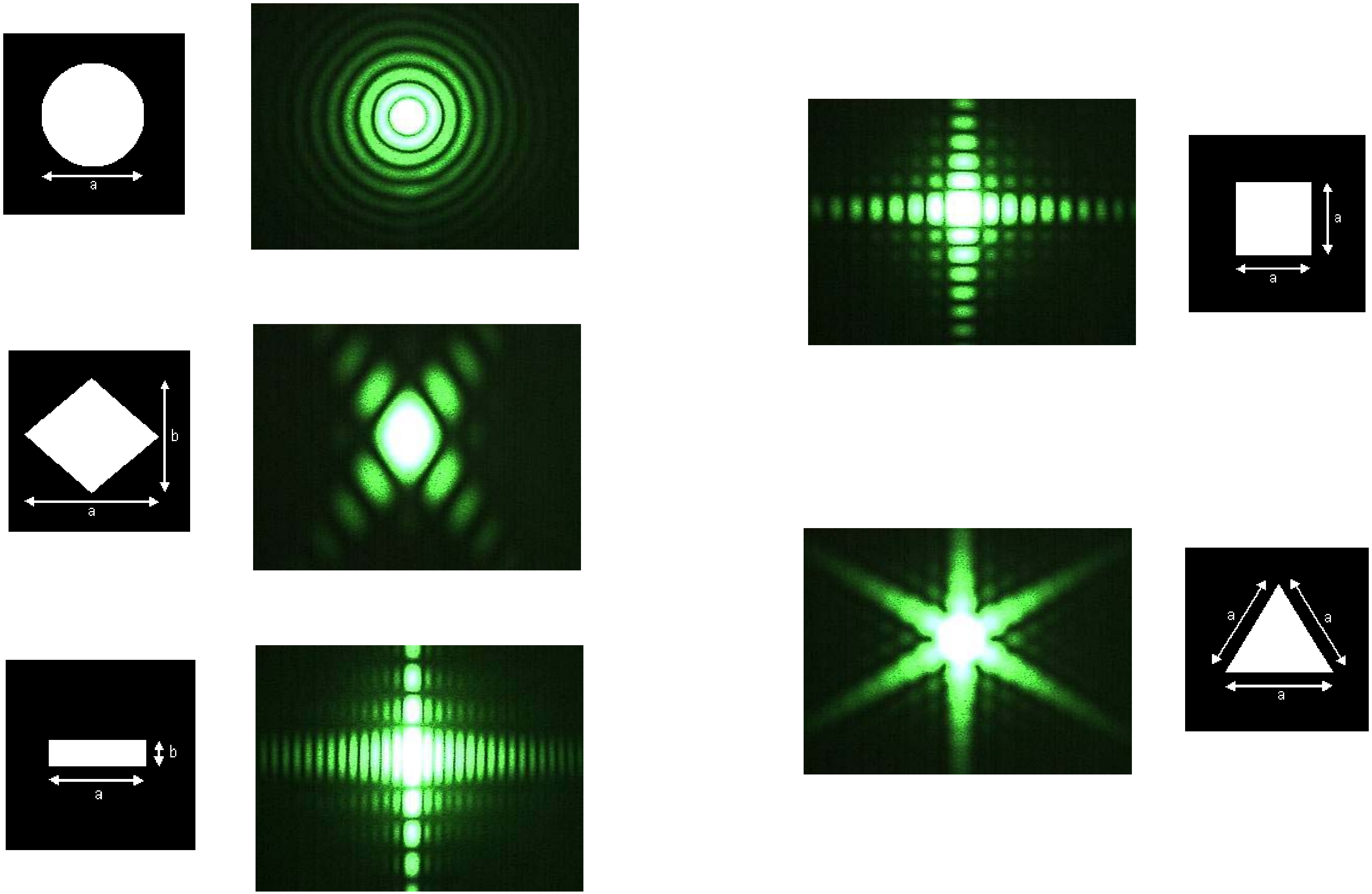
$$d_{exp} = 4.3 \text{ mm}$$

$$I(k_x,k_y) \sim \left|\underline{E}(k_x,k_y)\right|^2 = 4 \cdot \left|\underline{E}_s(k_x,k_y)\right|^2 \cdot \cos^2(\frac{c}{2}k_x) = 4 \cdot I_s(k_x,k_y) \cdot \cos^2(\frac{c}{2}k_x)$$

Intensity distribution of single form „circle“

Modulation in x-direction

Visualize basic laws of FT – Symmetry

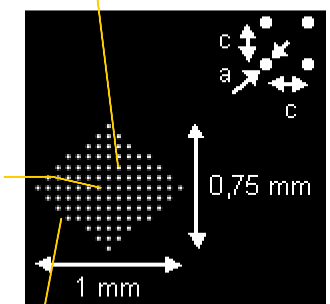


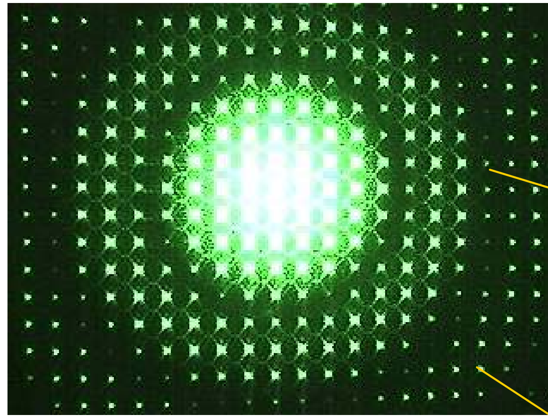
Structure of diffraction pattern

Single form circle

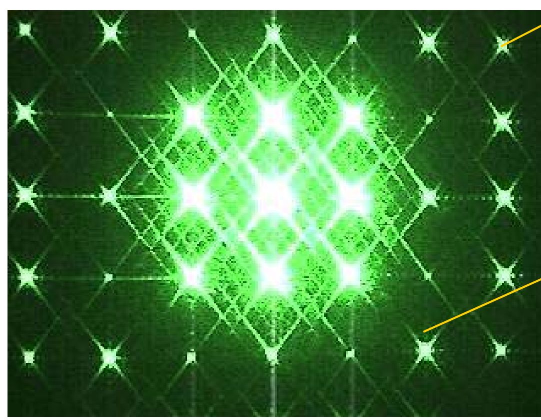
Quadratic lattice

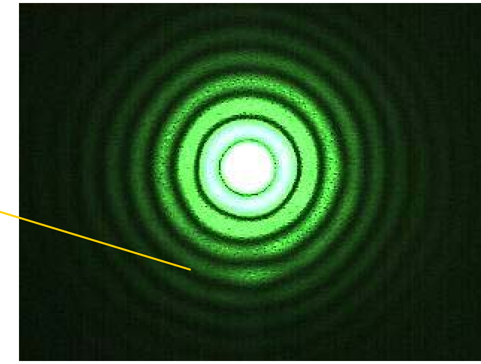
Boundary rhombus





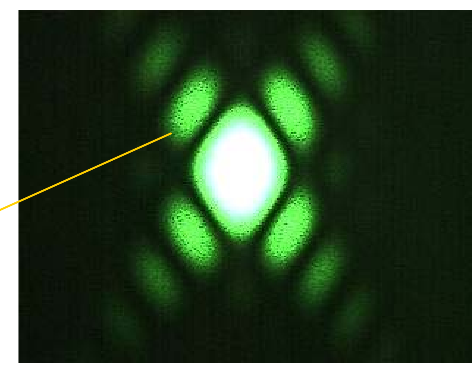
Zoom in





Diffraction pattern of single form circle

Diffraction pattern of quadratic lattice

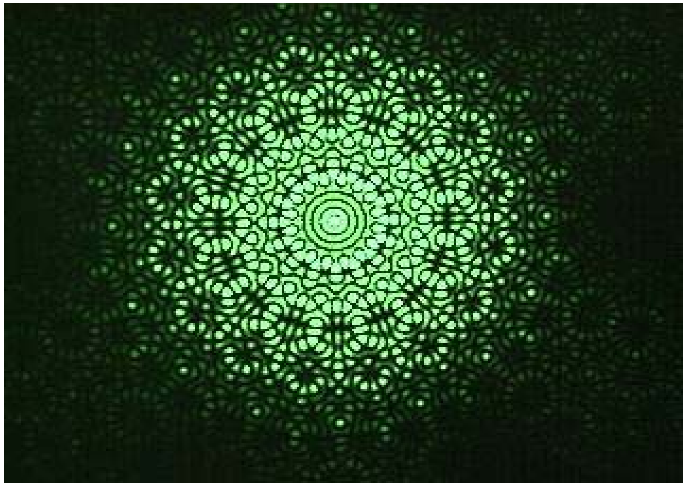


Diffraction pattern of boundary rhombus

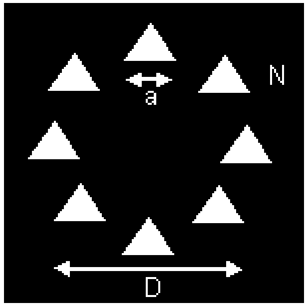
Conclusion

- This RCL works well qualitatively/quantitatively
- Experimental results agree with theoretical predictions
- Diffracting objects ($\approx$ 100) made by electron beam lithography
- Visualize mathematical relations of FT
- Further studies

- symmetry considerations of diffracting object and diffraction pattern
- from single forms to complex structures
- transition from ordered to disordered forms



Diffraction pattern of what?



a = 15 μm
D = 240 μm
N = 16